Stage-based Evaluation Plan with Robust Optimization

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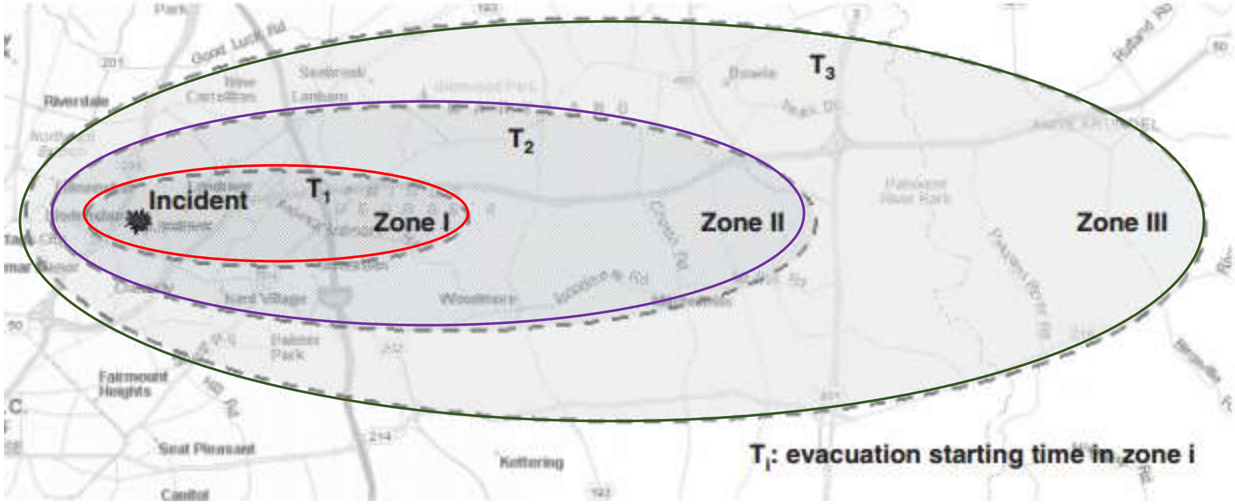
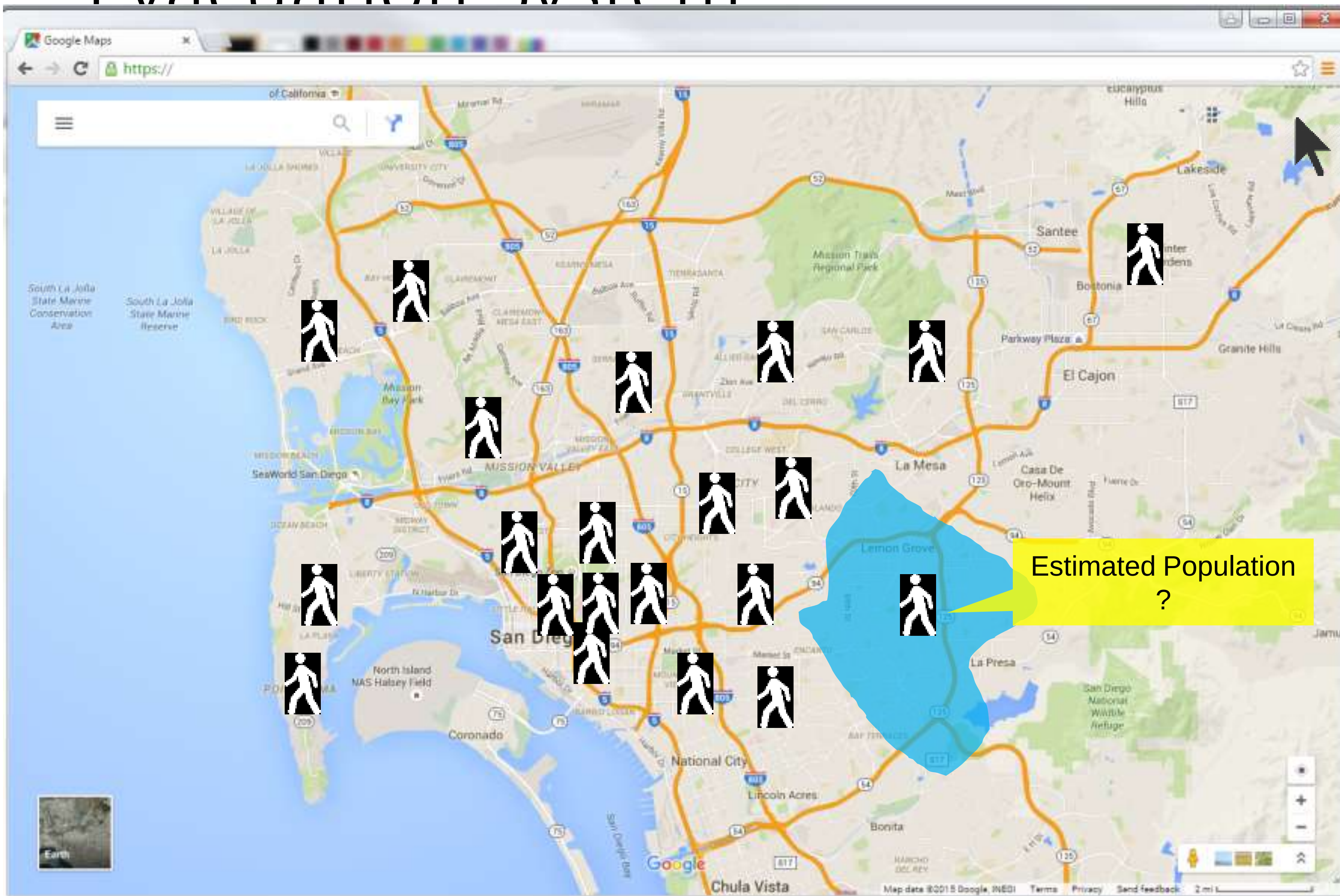
Staged Evacuation Concept



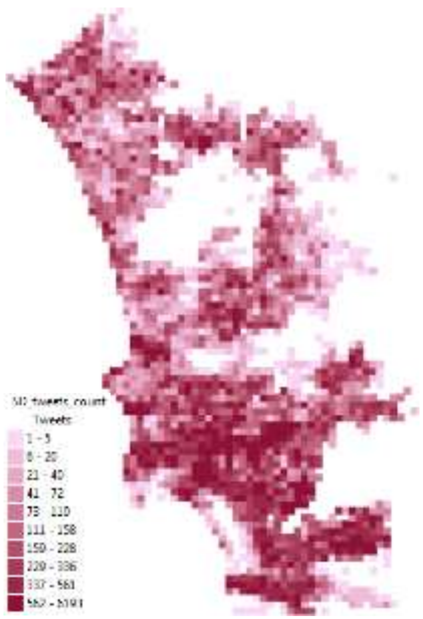
- The entire network is first divided into several zones on the basis of the predicted evolution of the emergency impacts;
- A zone closer to the incident location generally has a shorter safety time window and suffers more sever impacts;
- With these concerns, this study uses the staged evacuation strategy to determine the time to issue an evacuation order in each zone.

Evacuation System

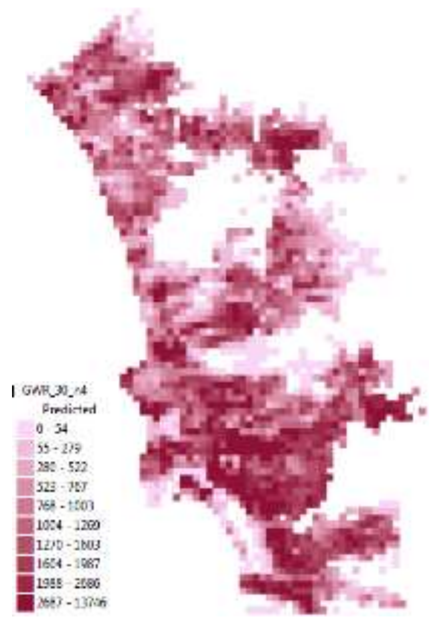


Stage-based Robust Evacuation

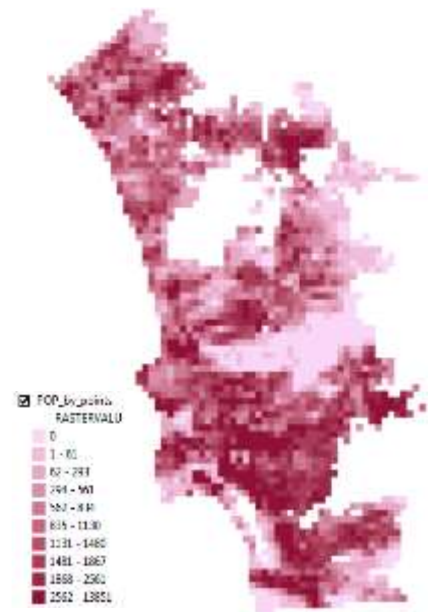
- In this project, we combine Twitter information with other data sources to estimate the hour-based population distribution.
- The estimated population distribution will be used as the demand input of evacuation system



Twitter message density

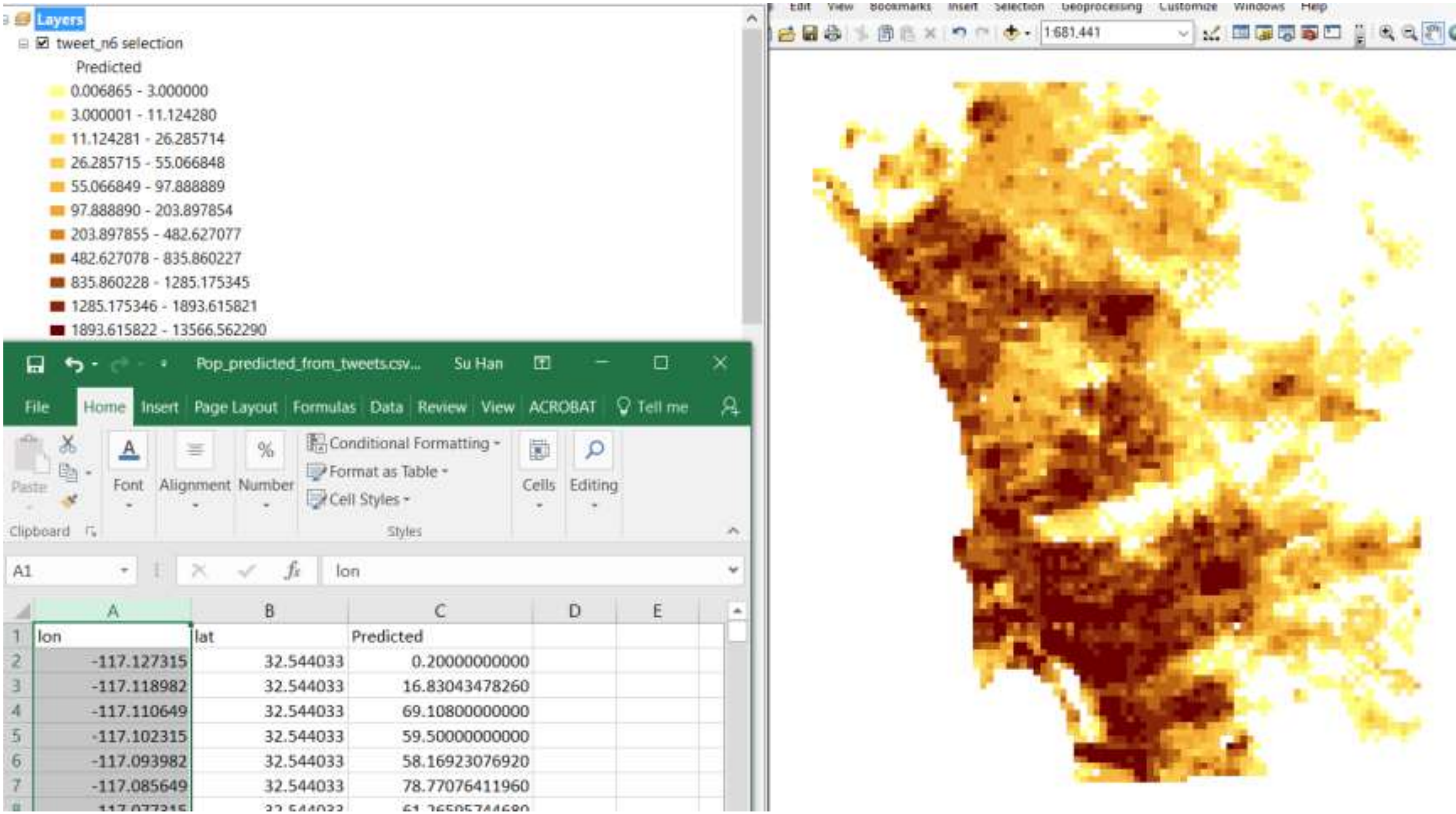


Estimated Population Density



Ground Truth

Stage-based Robust Evacuation



Demand loading pattern

- With the population density, the evacuation demand can be estimated with the car ownership information
- Then the next step is to distribute the evacuation demand among time with the logit-based function:

$$P(t) = \frac{1}{1 + \exp\left[-a\left(t - t_h\right)\right]}$$

where $P(t)$ is the cumulative percentage of the total demand loaded by time t after the evacuation order, and t_h and a are behavioral factors to define the half-loading time and response rate, respectively.

Robust Optimization Need

The uncertainty of input can caused by the estimates of:

- Population density
- # of persons/car
- The background traffic data

Base-model: Deterministic version

- To develop the robustness approach for modeling stage-based evacuation plan, the first task is to formulate its deterministic version.
- This study employed a cell-based optimization framework and formulated the stage-based evacuation problem with mixed-integer-linear-programming (MILP) technique

Base-model: Deterministic version

$$Min\ w_1 T_c^{\max} + w_2 T_e^{\max}$$

s.t.

$$\theta_i(t) \le M x_i(t) \tag{2}$$

$$\theta_i(t) \ge x_i(t) / M \tag{3}$$

$$\Delta t \sum_t \theta_i(t) \le T_c^{Max}; \quad i \in \sum_z \Omega_z \tag{4}$$

$$\Delta t \sum_t \theta_i(t) \le T_e^{Max}; \quad i \in \Omega - \Omega_D \tag{5}$$

$$\sum_{t=0}^{LS_z} \delta_z(t) = 1 \tag{6}$$

$$y_{ra}(t) = \delta_z(a) d_r(t); \qquad a = 1, 2, \ldots, LS_z,\ r \in \Omega_O \tag{7}$$

$$y_{rw}(t) = \delta_z(0) d_r(t); \qquad r \in \Omega_O \tag{8}$$

$$y_{a+1,a}(t) = x_{a+1}(t); \qquad a = 1, 2, \ldots, LS_z \tag{9}$$

$$y_{1,w}(t) = x_1(t) \tag{10}$$

$$d_r(t) = P_r(t) \frac{A_r}{p_r} \tag{11}$$

$$x_i(t+1) = x_i(t) + \sum_{k \in \Gamma^{-1}(i)} y_{ki}(t) - \sum_{j \in \Gamma(i)} y_{ij}(t) \tag{12}$$

$$\sum_{j \in \Gamma(i)} y_{ij}(t) \le Q_i(t) \tag{13}$$

$$\sum_{j \in \Gamma(i)} y_{ij}(t) \le x_i(t) \tag{14}$$

$$\sum_{k \in \Gamma^{-1}(i)} y_{ki}(t) \le Q_i(t) \tag{15}$$

$$\sum_{k \in \Gamma^{-1}(i)} y_{ki}(t) \le N_i(t) - x_i(t) - b_i(t) \tag{16}$$

$$\sum_t \sum_{k \in \Gamma^{-1}} y_{kq}(t) = \sum_{r \in \Omega_O} A_r / p_r; \qquad q \in \Omega_D \tag{17}$$

Uncertainty set and robust model

The main problem of the deterministic model is that many variables are uncertain in real-world applications:

- Evacuation Demand A_r and $A_r \in [A_{r\_min}, A_{r\_max}]$
- Background traffic $b_j(t)$ and $b_j(t) \in [b_j(t)_{\_min}, b_j(t)_{\_max}]$

In strict robustness, the models always aim to explore the optimal solution that can offer best performance under the "worst case":

$$\min \Psi(w_1 T_c^{\max} + w_2 T_e^{\max})\{WC : A_r = A_{r\_\max}; b_j(t) = b_j(t)_{\max}\}$$

Recoverable Robust Model

However, considering the low-probability of occurring the worst-case in practice, we aim to find another robust solution which:

- can provide acceptable performance under the “worst-case”
- can be easily recovered to other normal cases

Then the recoverable robust optimization model can be formulated with a multi-objective framework:

$$\min\ (w_1 T_c^{\max} + w_2 T_e^{\max})\{A_r = A_{r\_\max}; b_j(t) = b_j(t)_{\max}\}$$

$$\min\{\sum_i \sum_t (x_i(t) - x_i(t)\_\xi);\ \forall \xi \in \Psi\}$$

Solution Algorithm

- Step 1: generate a scenario set Θ based on the uncertainty intervals;
- Step 2: Solve the deterministic model for each scenario $\xi \in \Theta$ to obtain the routing plan $\{x_i(t)\_\xi\}$;
- Step 3: Solve the sub-problem:

$$\min\{\sum_i \sum_t (x_i(t) - x_i(t)\_\xi);\ \forall \xi \in \Theta\}$$

- Step 4: Given the obtained $\{x_i(t)\}$; solve the sub-problem to find the worst case under such condition:

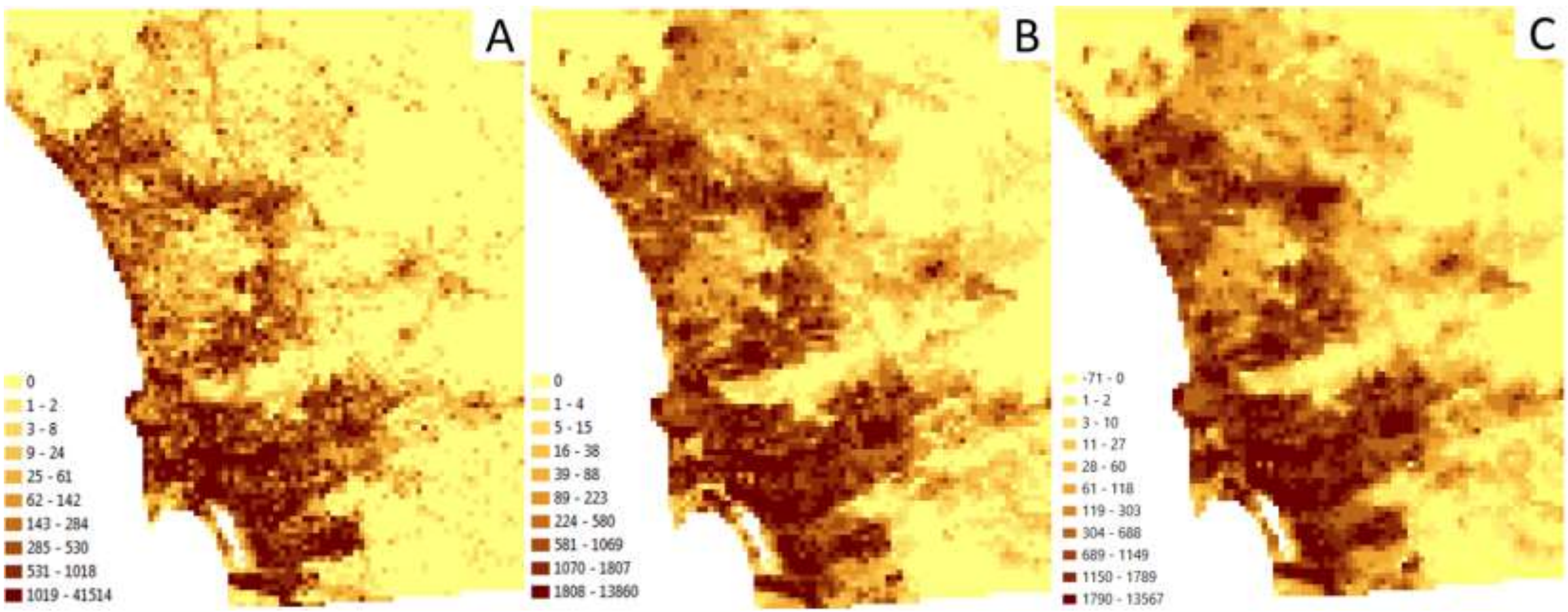
$$arcMax\{w_1 T_c^{\max} + w_2 T_e^{\max}\}\_WC$$

- Step 5: Add the scenario obtained from step 4 to the Θ;
- Step 6: If stop criteria (i.e., results converged or maximum number of iterative reached) is not met, go back to Step 2; otherwise stop the search.

Case Study

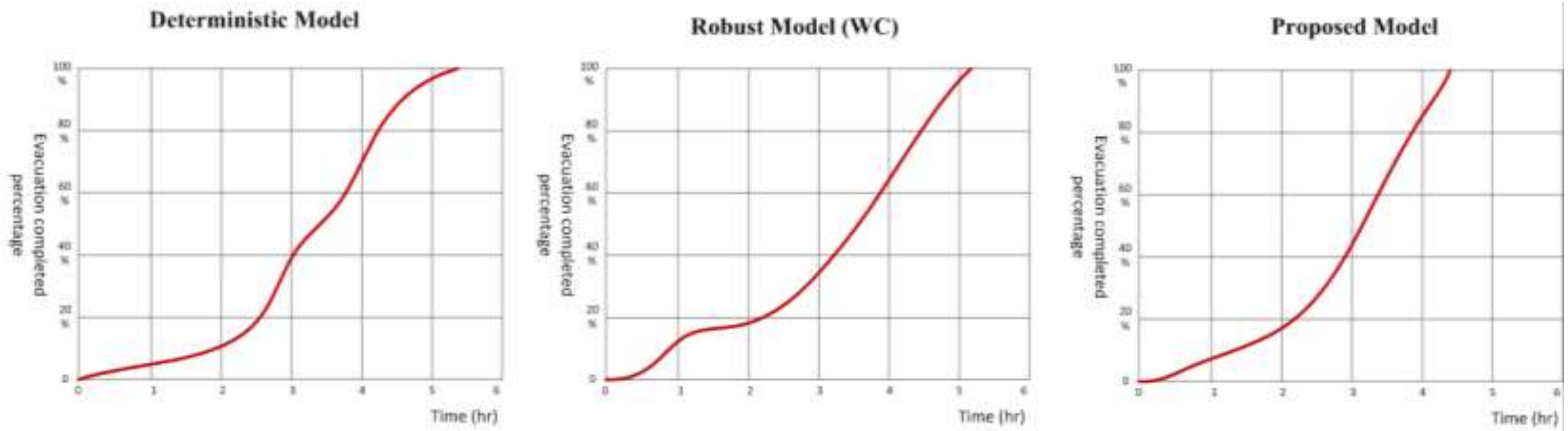
- Assuming a wildfire incident happened in Zone 24, at 3PM of a weekday, and will spread out to its nearby zones, 3 and 4 in four hours.
- The safe destinations are set in Zone 16, 13, 10, 8, 6, and 7.
- All residents within Zone 3, 4, and 24 must evacuate to the safe destinations and all the three zones must be cleared before the reach of wildfire impact

Case Study

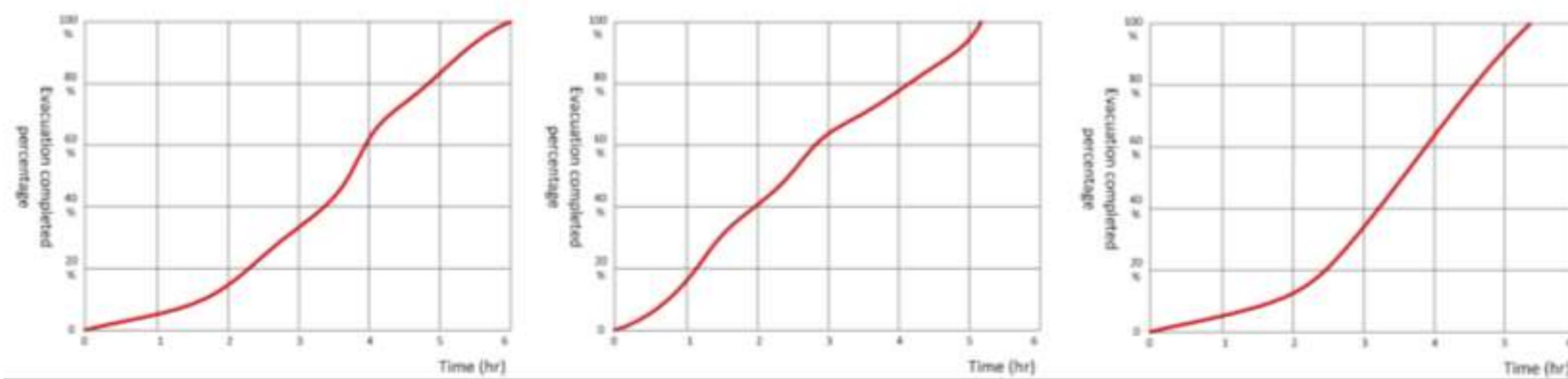


The distribution of population density

Case Study



The time-dependent evacuation completion rate under the random case



The time-dependent evacuation completion rate under the worst-case

Questions?